



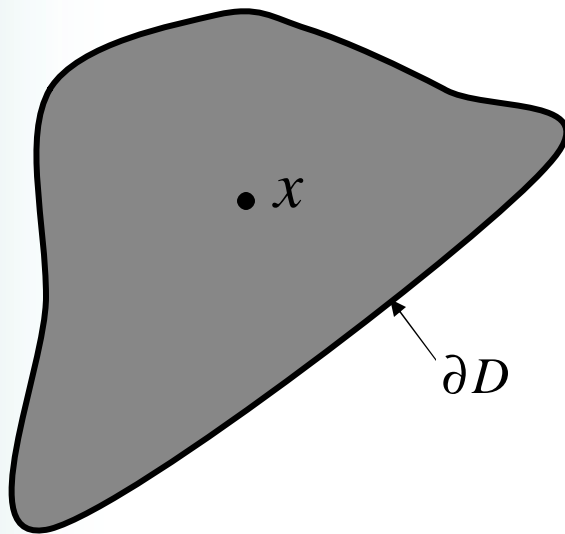
The Method Moments: The Perfect ClearSpeed Application

ClearSpeed Technology plc



What is the Method of Moments?

Solution on the boundary is sufficient

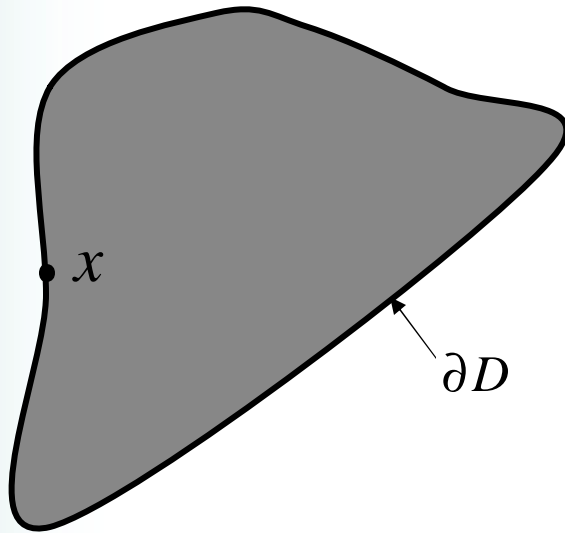


- **The field (F) at any point can be calculated from**
 - the field on the boundary (F)
 - the gradient of the field normal to the boundary ($\frac{\partial F}{\partial n}$)
 - a Green's function (G).

$$F(x) + \int_{\partial D} F \frac{\partial G}{\partial n} ds = \int_{\partial D} G \frac{\partial F}{\partial n} ds$$

Also works for boundary points

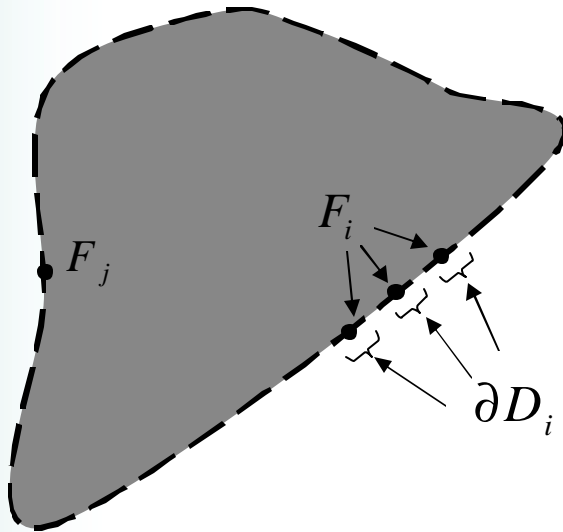
- **Small change to formula**



$$\frac{1}{2} F(x) + \int_{\partial D} F \frac{\partial G}{\partial n} ds = \int_{\partial D} G \frac{\partial F}{\partial n} ds$$

Discrete version for computation

- Divide the boundary into m elements



$$\frac{1}{2} F_j + \sum_{i=1}^m \int_{\partial D_i} F_i \frac{\partial G_j}{\partial n_i} ds_i = \sum_{i=1}^m \int_{\partial D_i} G_j \frac{\partial F_i}{\partial n_i} ds_i$$

$$j = 1 \cdots m$$

Matrix formulation

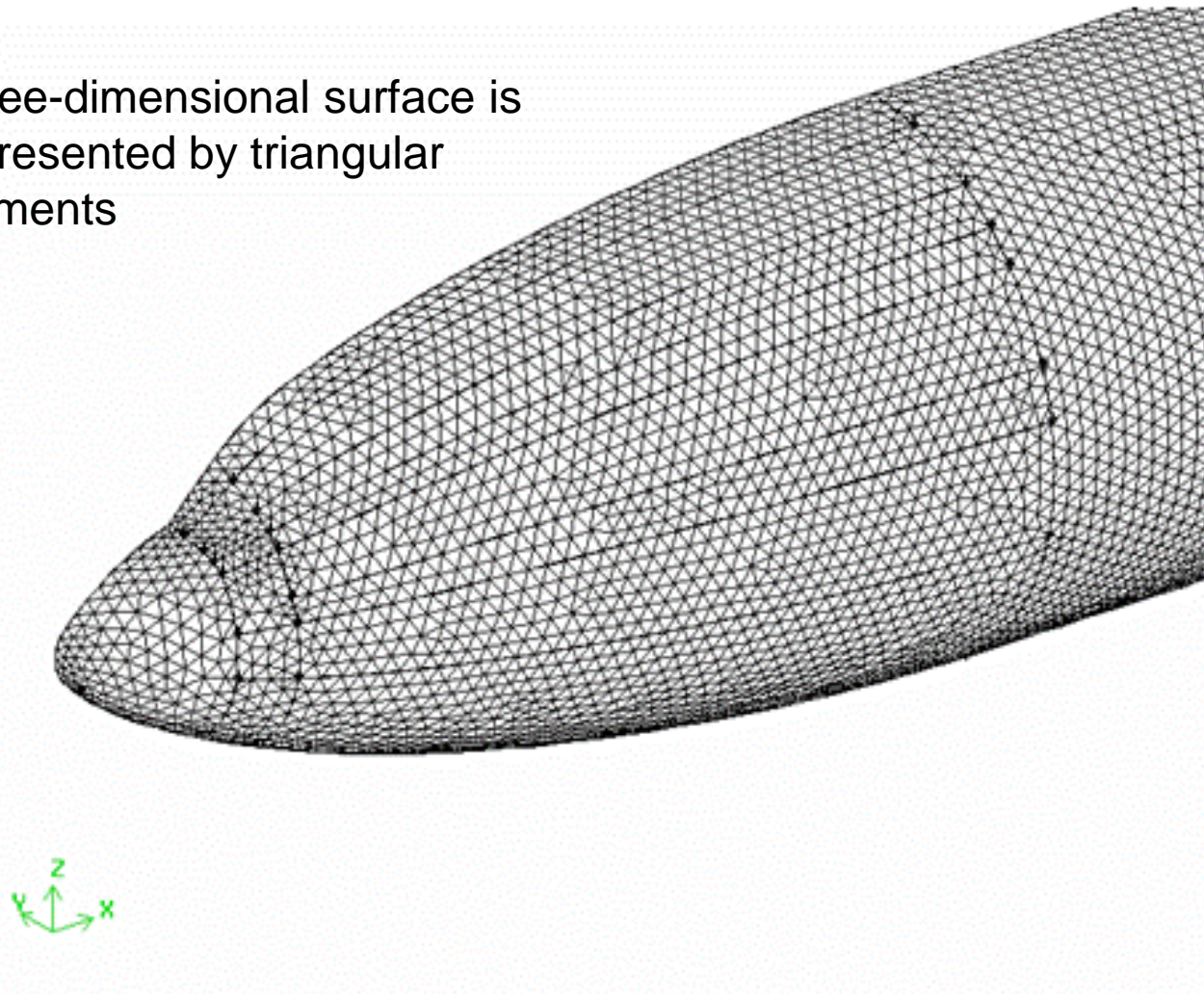
- Rewrite as matrix of equations (four equations in this example)

$$\begin{bmatrix} \int \frac{\partial G_1}{\partial n} ds + \frac{1}{2} & \int \frac{\partial G_1}{\partial n} ds & \int \frac{\partial G_1}{\partial n} ds & \int \frac{\partial G_1}{\partial n} ds \\ \int \frac{\partial G_2}{\partial n} ds & \int \frac{\partial G_2}{\partial n} ds + \frac{1}{2} & \int \frac{\partial G_2}{\partial n} ds & \int \frac{\partial G_2}{\partial n} ds \\ \int \frac{\partial G_3}{\partial n} ds & \int \frac{\partial G_3}{\partial n} ds & \int \frac{\partial G_3}{\partial n} ds + \frac{1}{2} & \int \frac{\partial G_3}{\partial n} ds \\ \int \frac{\partial G_4}{\partial n} ds & \int \frac{\partial G_4}{\partial n} ds & \int \frac{\partial G_4}{\partial n} ds & \int \frac{\partial G_4}{\partial n} ds + \frac{1}{2} \end{bmatrix} \begin{bmatrix} F_1 \\ F_2 \\ F_3 \\ F_4 \end{bmatrix} = \begin{bmatrix} \int \frac{G_1}{\partial D_1} ds & \int \frac{G_1}{\partial D_1} ds & \int \frac{G_1}{\partial D_1} ds & \int \frac{G_1}{\partial D_1} ds \\ \int \frac{G_2}{\partial D_2} ds & \int \frac{G_2}{\partial D_2} ds & \int \frac{G_2}{\partial D_2} ds & \int \frac{G_2}{\partial D_2} ds \\ \int \frac{G_3}{\partial D_3} ds & \int \frac{G_3}{\partial D_3} ds & \int \frac{G_3}{\partial D_3} ds & \int \frac{G_3}{\partial D_3} ds \\ \int \frac{G_4}{\partial D_4} ds & \int \frac{G_4}{\partial D_4} ds & \int \frac{G_4}{\partial D_4} ds & \int \frac{G_4}{\partial D_4} ds \end{bmatrix} \begin{bmatrix} dF_1 \\ dF_2 \\ dF_3 \\ dF_4 \end{bmatrix}$$

- Evaluate coefficients and solve set of linear equations

Three-dimensional version

Three-dimensional surface is represented by triangular elements





The Method of Moments for Computational Electromagnetics

Radar cross-section



Lockheed
F-117A Nighthawk



Northrop Grumman
B-2 Spirit

Antenna design



Helical antenna



Short-wave transmitter

Algorithm for solving Maxwell's equations

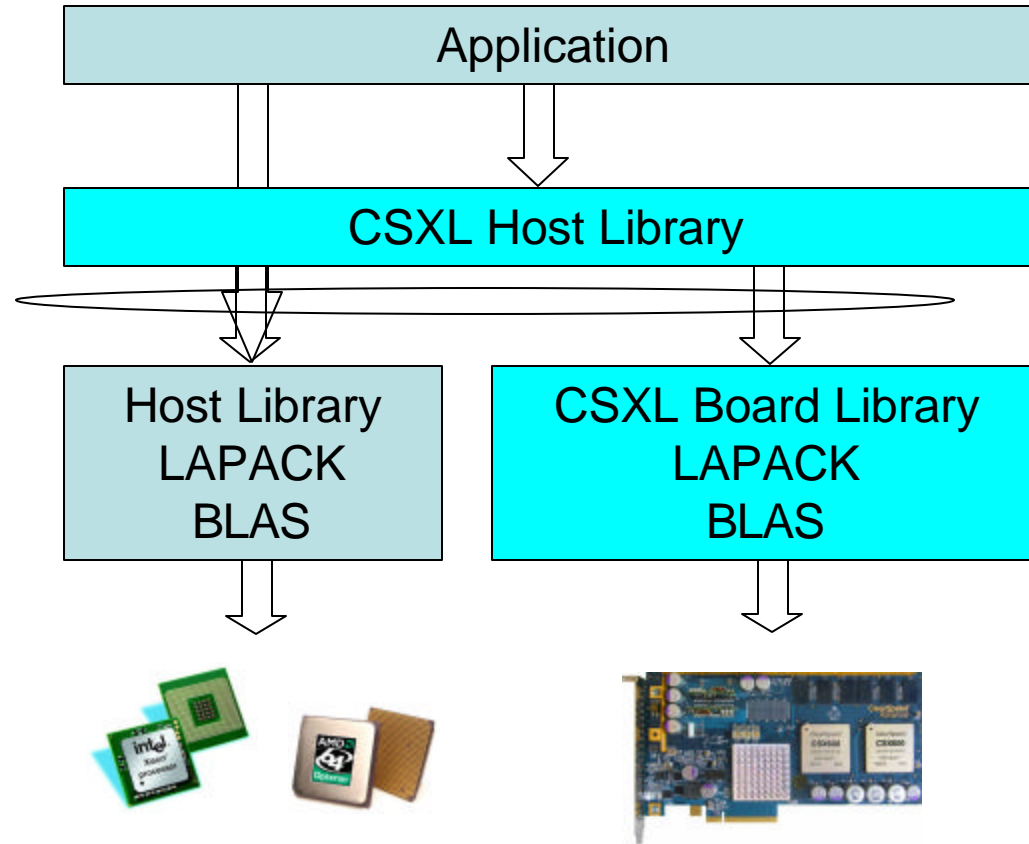
- **Build triangular mesh representation of geometry of target**
- **Calculate coefficients of equations using numerical integration**
 $O(n^2)$ operations
- **Perform LU-decomposition of matrix**
 $O(n^3)$ operations
- **Back-substitution with multiple right-hand sides**
 $O(n^2)$ operations
- **Overall computation dominated by LU-decomposition — can be weeks of computation**



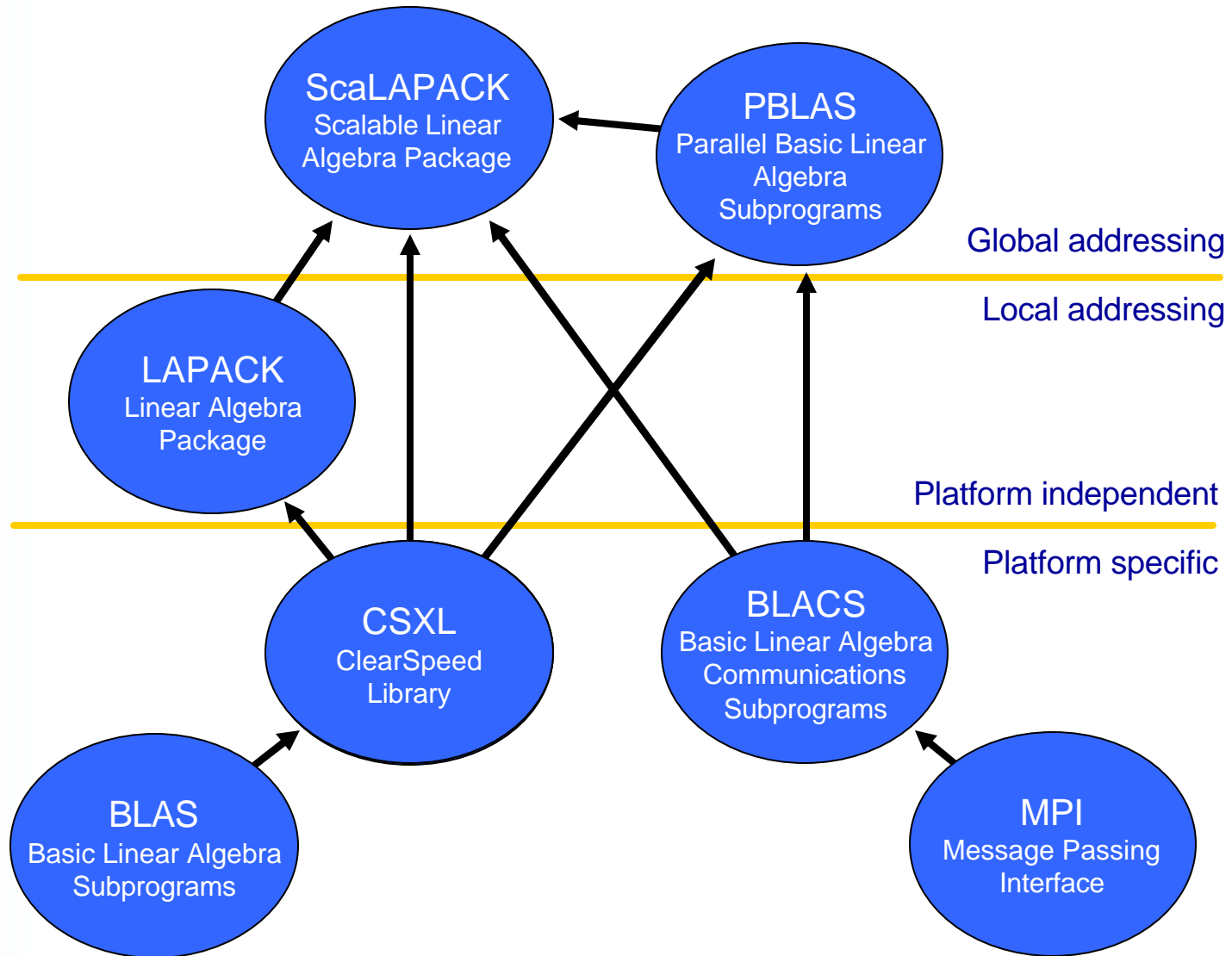
How to Accelerate the Method of Moments

Plug and play acceleration with libraries

Automatically
select
optimum path



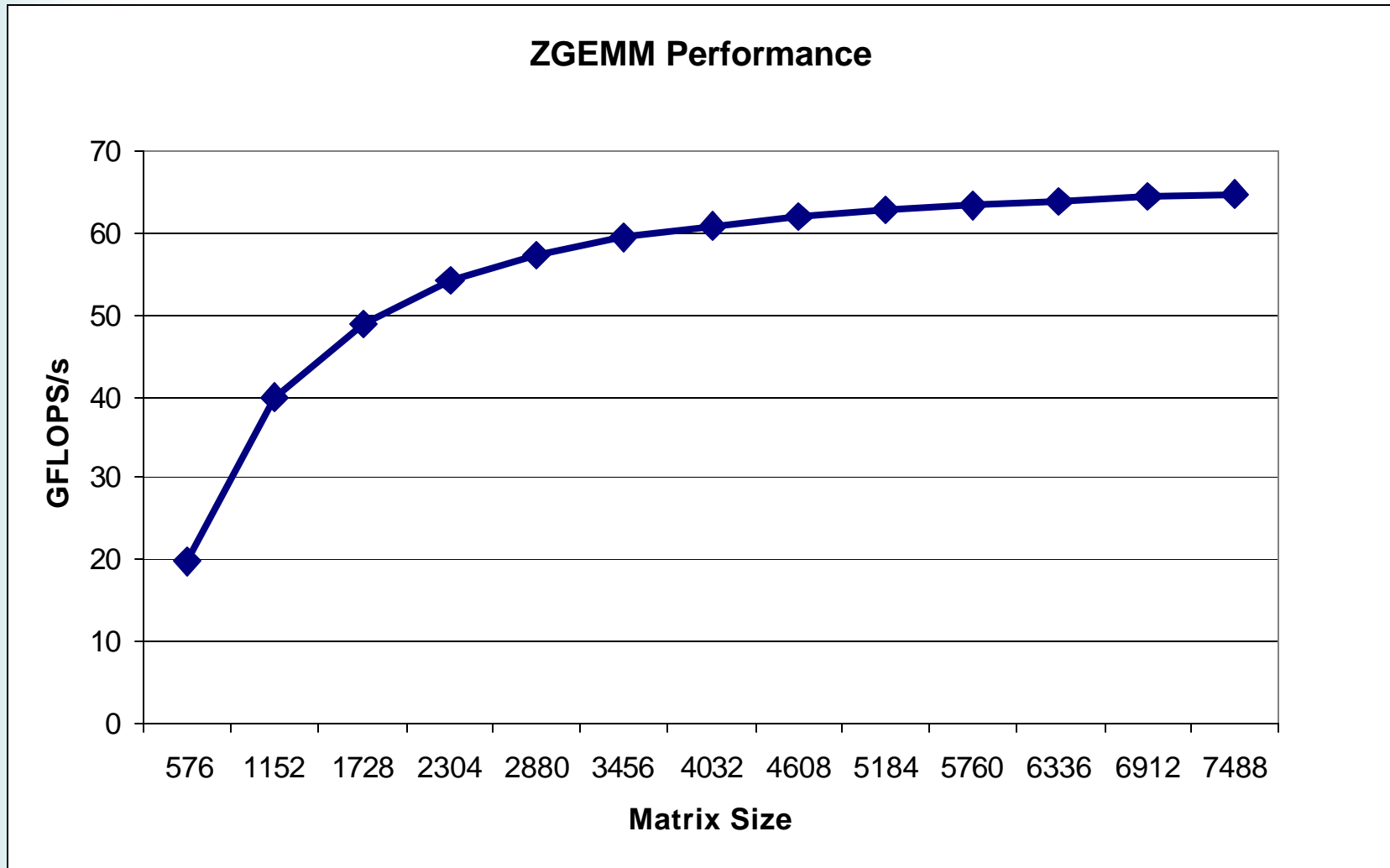
Linear algebra software stack





How Much Acceleration?

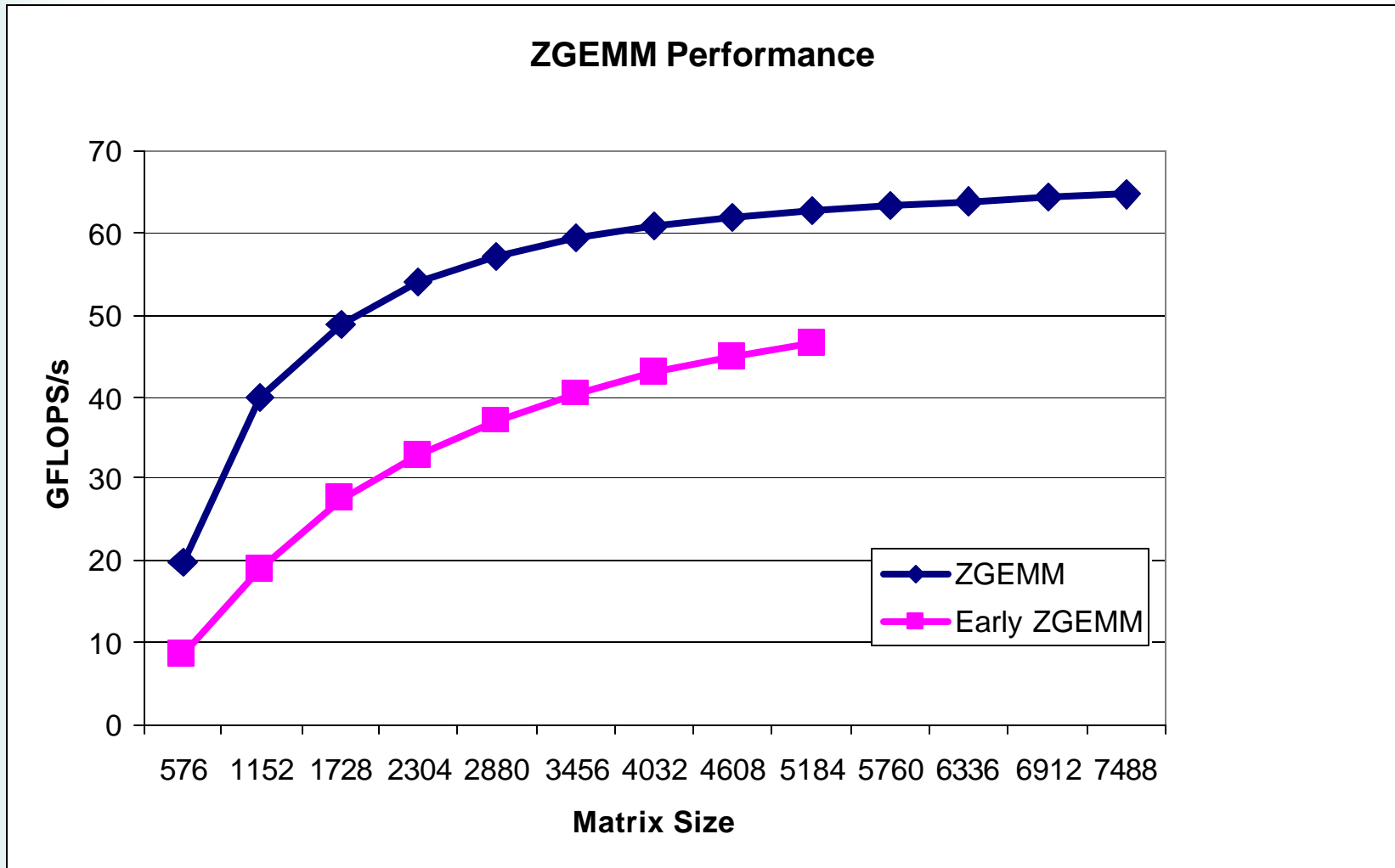
ZGEMM performance



Complete linear solver implementation

- **Host system**
 - Intel Xeon quad-core *Clovertown*, 2 GHz clock frequency
Only one core used for these tests
 - 16 Gbytes memory
- **Accelerator**
 - ClearSpeed Advance e620
- **Software stack**
 - ScaLAPACK, PBLAS, BLACS from Netlib
 - LAPACK, BLAS from AMD Core Math Library (ACML)
 - MPI from MPICH or MVAPICH
 - Fortran and C compilers from Intel
 - Early implementation of ClearSpeed ZGEMM with reduced performance

Early ZGEMM performance



Problem definition and performance

	Host	Accelerator
Number of equations	15552	15552
Block size	256	576
% time spent in ZGEMM	92%	65%
Run time	1998 seconds	889 seconds

Further improvements

- **Host and accelerator together**
- **Improved ZGEMM**
- **Multi-core host and multiple accelerators**
- **ClearSpeed Advance e710 accelerator**

*ClearSpeed*TM

